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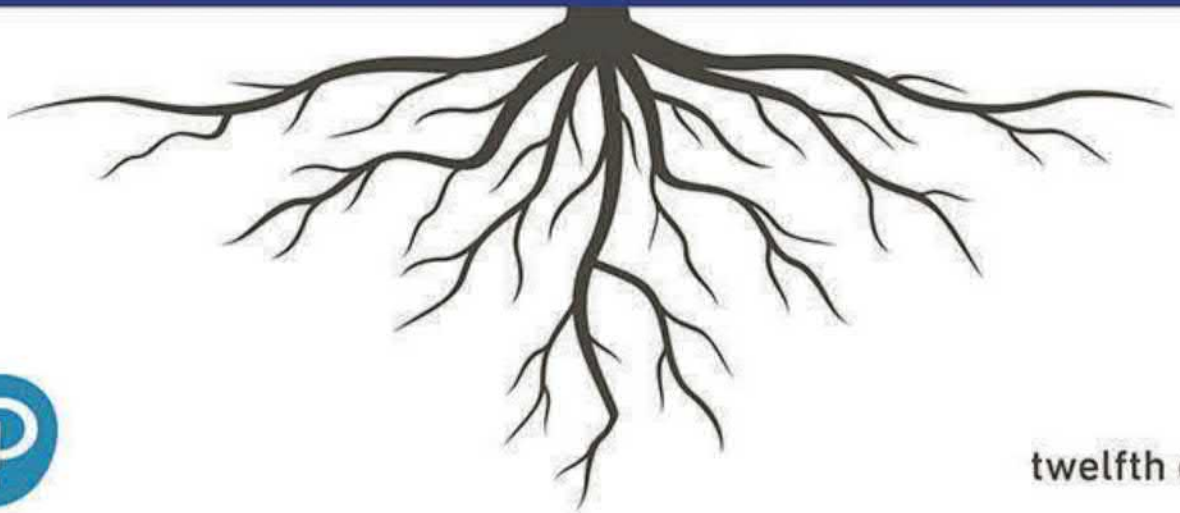
TRIGONOMETRY



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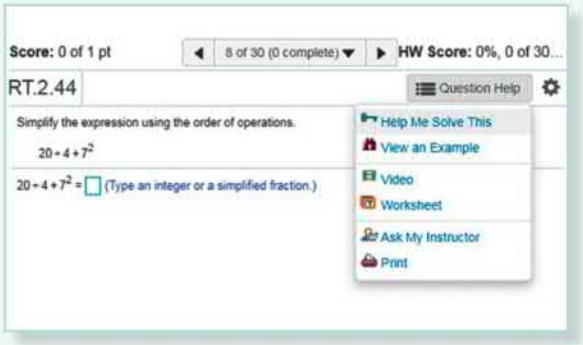

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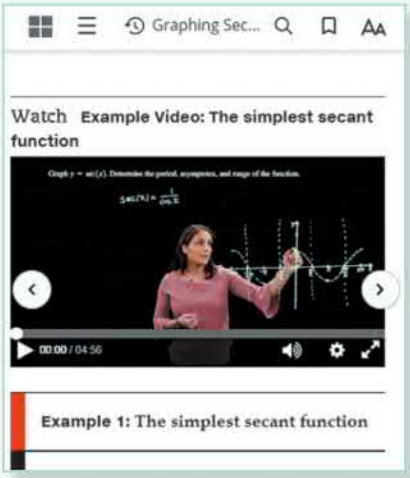

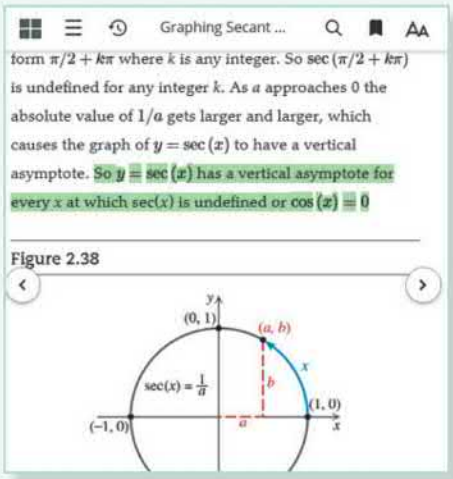

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Trigonometry

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Trigonometry

TWELFTH EDITION

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This text is dedicated to you—the student. We hope that it helps you achieve your goals. Remember to show up, work hard, and stay positive. Everything else will take care of itself.

The Lial Author Team

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Preface

WELCOME TO THE 12TH EDITION

In the twelfth edition of *Trigonometry,* we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, corequisite, flipped, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyLab Math.

In this edition, we have drawn on the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability as we continually strive to make math understandable for students, updates to our extensive list of applications and real-world mathematics problems, use of color in displays and side comments, and coordination of exercises and their related examples.

The authors understand that teaching and learning mathematics today can be a challenging task. Some students are prepared for the challenge, while other students require more review and supplemental material. This text is written so that trigonometry students with varying abilities and backgrounds will all have an opportunity for a successful learning experience.

The Lial team believes this to be our best edition of *Trigonometry* yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional texts in this series are as follows.

College Algebra, Thirteenth Edition
College Algebra and Trigonometry, Seventh Edition
Precalculus, Seventh Edition

HIGHLIGHTS OF NEW CONTENT

- **New Chapter R** provides a thorough review of the basic algebraic concepts that trigonometry students need in order to succeed. Topics include real number operations and properties, rules for exponents, operations on polynomials and factoring, rational and radical expressions, solving equations and inequalities, circles, functions, domain, range, and graphing techniques.

  Instructors may choose to cover review topics from **Chapter R** at the beginning of their course or to insert these topics as-needed in a just-in-time fashion. Either way, students who are under-prepared for the demands of trigonometry, as well as those who need a quick review, will benefit from the material contained here. It is our hope that this chapter, which also includes a **comprehensive end-of-chapter Test Prep,** would serve as a valuable resource throughout all course types for students and instructors alike.

- The **exercise sets** were a key focus of this revision, and **Chapters 3 and 4** are among the chapters that have benefitted. Specifically, **Section 3.2 Applications of Radian Measure** has additional exercises devoted to finding arc length and area of a sector. **Section 3.4 Linear and Angular Speed** now contains new applications of linear and angular speed. **Chapter 4** includes updated real data applications and additional ***Connecting Graphs with Equations*** exercises, as well as new applications of harmonic motion.

- Proofs of identities in **Chapter 5** now feature a drop-down style for increased clarity and student understanding. Based on reviewer requests, **Section 6.4 Equations Involving Inverse Trigonometric Functions** includes new exercises in which solutions of inverse trigonometric equations are found.
- In response to reviewer suggestions, **Section 7.5 Algebraically Defined Vectors and the Dot Product** has new exercises on finding the angle between two vectors, determining magnitude and direction angle for a vector, and identifying orthogonal vectors. Additionally, **Chapter 8** contains new exercises requiring students to graph polar and parametric equations and give parametric representations of plane curves.

FEATURES OF THIS TEXT

SUPPORT FOR LEARNING CONCEPTS

We provide a variety of features to support students' learning of the essential topics of trigonometry. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student understands the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as sports, biology, astronomy, geology, music, and environmental studies.
- **Function Boxes** Special function boxes offer a comprehensive, visual introduction to each type of trigonometric function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyLab Math.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to promote visual appeal. Guided Visualizations with accompanying exercises and explorations are available and assignable in MyLab Math.
- **Cautions and Notes** Text that is marked CAUTION warns students of common errors, and NOTE comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.

- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although ***this technology is completely optional and can be omitted without loss of continuity.*** We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon .

SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master trigonometry. The extensive exercise sets provide ample opportunity for practice and increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes mastery of the concepts and reduces the need for rote memorization.

- **Concept Preview** Each exercise set begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts in each section. These exercises are assignable in MyLab Math and provide support especially for hybrid, online, and flipped courses.
- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator exercises , and multiple-choice, matching, true/false, and completion exercises. Those marked ***Concept Check*** focus on conceptual thinking. ***Connecting Graphs with Equations*** exercises challenge students to write equations that correspond to given graphs. Video solutions for select problems are available in MyLab Math.
- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyLab Math.

SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found both within the chapters and at the ends of chapters. Quizzes interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter Test Preps provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyLab Math.
- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods.
- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms,** a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test.** The Test Prep, Review Exercises, and Chapter Test are assignable in MyLab Math.

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MyLab Math

Pearson MyLab

MyLab Math for Trigonometry 12e

(access code required)

MyLab Math is tightly integrated with author style, offering a range of author-created resources, to give students a consistent experience.

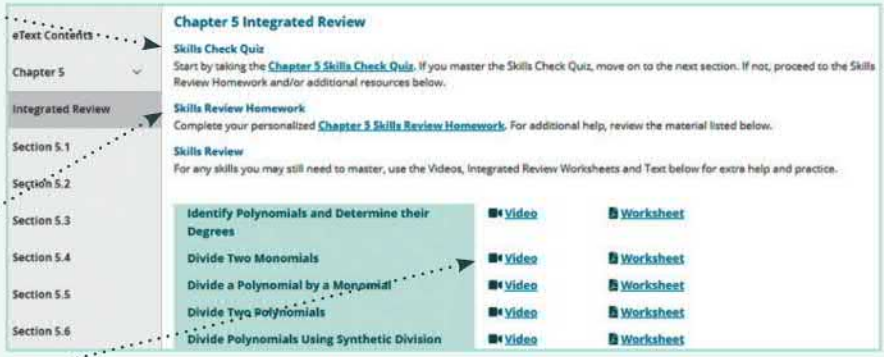
Preparedness

Preparedness is one of the biggest challenges in many math courses. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review as needed.

Integrated Review in MyLab Math

Integrated Review can be used in corequisite courses or simply to help students who enter a course without a full understanding of prerequisite skills and concepts. Premade, editable Integrated Review assignments are available to assign in the Assignment Manager. Integrated Review landing pages (shown below) are visible by default at the start of most chapters, providing objective-level review.

- Students begin each chapter by completing a Skills Check to pinpoint which topics, if any, they need to review.
- Personalized review homework provides extra support for students who need it on just the topics they didn't master in the preceding Skills Check.



- Additional review materials, including worksheets and videos, are available.

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*Get the **most** out of* MyLab Math

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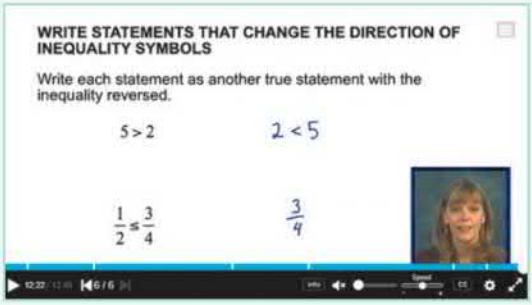
Updated! Videos

Updated videos cover all topics in the text to support students outside of the classroom. **Quick Review** videos cover key definitions and procedures. **Example Solution** videos offer a detailed solution process for every example in the textbook.



Chapter 2 Graphs and Functions

2.1 Rectangular Coordinates and Graphs

Ordered Pairs

CLASSROOM EXAMPLE 1

The Rectangular Coordinate System

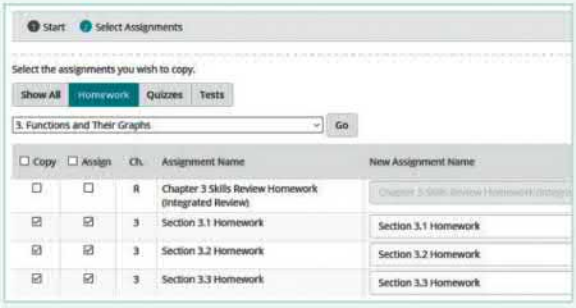
Updated! MyNotes and MyClassroomExamples

MyNotes give students a note-taking structure to use while they read the text or watch the MyLab Math videos. **MyClassroomExamples** offer structure for notes taken during lecture and are for use with the ClassroomExamples found in the Annotated Instructor Edition.

Both sets of notes are available in MyLab Math and can be customized by the instructor.

New! Enhanced Sample Assignments

Author Callie Daniels makes course set-up easier by giving instructors a starting point for each section. Following Callie's best practices in the classroom, Enhanced Sample Assignments maximize students' performance.



- **Section Prep Assignments** include Example Videos with assessment questions. This assignment pairs with **MyNotes**. Students actively participate while taking notes from the Example Video and then work the related exercises.
- **Section Homework** includes author-selected problems and increases in difficulty.
- **Cumulative Review Homework Assignments** draw from section homework questions covered to that point in the course—helping students prepare for a final exam.

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Resources for Success

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Instructor Resources

Online resources can be downloaded at pearson.com/mylab/math or from **www.pearson.com**.

Annotated Instructor's Edition

ISBN: 0135926033 / 9780135926031
Answers are included on the same page beside the text exercises where possible for quick reference. Helpful Teaching Tips and Classroom Examples are also provided.

Online Instructor's Solution Manual

By Beverly Fusfield
Provides complete solutions to all text exercises

Online Instructor's Testing Manual

Includes diagnostic pretests, grouped by section, with answers provided

Testgen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides and Classroom Example PowerPoints

- The PowerPoint Lecture Slides feature presentations written and designed specifically for this text, including figures and examples from the text.
- Classroom Example PowerPoints include fully worked-out solutions to all Classroom Examples.

Learning Catalytics™

With MyLab Math, instructors and students have access to Learning Catalytics, which instructors can use to generate class discussion, guide lectures, and actively engage students. Prebuilt Learning Catalytics questions have been created specifically for this text. Simply search the tag "LialTrigonometry" within the Learning Catalytics Question Library.

Student Resources

Additional resources enhance student success.

Student's Solution Manual

By Beverly Fusfield
Provides detailed solutions to all odd-numbered text exercises

Video Lectures

- Quick Reviews cover key definitions and procedures from each section.
- Example Solutions walk students through the detailed solution process for every example in the textbook.

MyNotes with Integrated Review Worksheets

MyNotes offer structure for students as they watch videos or read the text. These are available as a printed supplement and in MyLab Math.

- Includes textbook examples along with ample space for students to write solutions and notes
- Includes key concepts along with prompts for students to read, write, and reflect on what they have just learned
- **Customizable**—instructors can add their own examples or remove examples that are not covered in their course.

Integrated Review Worksheets prepare students for the Trigonometry material.

- Includes key terms, guided examples with ample space for students to work, and references to extra help in MyLab Math

MyClassroomExamples

- Available in MyLab Math and offer structure for classroom lecture
- Includes Classroom Examples along with ample space for students to write solutions and notes
- Includes key concepts along with fill-in-the-blank opportunities to keep students engaged
- **Customizable**—instructors can add their own examples or remove Classroom Examples that are not covered in their course.

ACKNOWLEDGMENTS

We wish to thank the following individuals who provided valuable input into this edition of the text.

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As an author team, we are committed to providing the best possible trigonometry course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to math@pearson.com.

Margaret L. Lial
John Hornsby
David I. Schneider
Callie J. Daniels

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Trigonometry

TWELFTH EDITION

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R Algebra Review

Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

R.1 Basic Concepts from Algebra

R.2 Real Number Operations and Properties

R.3 Exponents, Polynomials, and Factoring

R.4 Rational Expressions

R.5 Radical Expressions

R.6 Equations and Inequalities

R.7 Rectangular Coordinates and Graphs

R.8 Functions

R.9 Graphing Techniques

R.1 Basic Concepts from Algebra

- Sets of Numbers
- Additive Inverses
- Absolute Value
- Inequality and Order on a Number Line

Sets of Numbers

A **set** is a collection of objects called the **elements,** or **members,** of the set. In algebra, the elements of a set are usually numbers. Set braces, { }, are used to enclose the elements. For example,

$$2 \text{ is an element of the set } \{1, 2, 3\}.$$

Because we can count the number of elements in the set $\{1, 2, 3\}$ and the counting comes to an end, it is a **finite set.**

In algebra, we refer to certain sets of numbers by name. The set

$$\{1, 2, 3, 4, \ldots\} \quad \text{Natural (counting) numbers}$$

is the **natural numbers,** or the **counting numbers.** The three dots (*ellipsis* points) show that the list continues in the same pattern indefinitely. We cannot list all of the elements of the set of natural numbers, so it is an **infinite set.**

Including 0 with the set of natural numbers gives the set of **whole numbers.**

$$\{0, 1, 2, 3, 4, \ldots\} \quad \text{Whole numbers}$$

The set containing no elements is the **empty set,** or **null set,** usually written $\emptyset$. For example, the set of whole numbers less than 0 is $\emptyset$.

CAUTION Do not write $\{\emptyset\}$ for the empty set. $\{\emptyset\}$ is a set with one element: $\emptyset$. Use the notation $\emptyset$ for the empty set.

To indicate that 2 is an element of the set $\{1, 2, 3\}$, we use the symbol $\in$, which is read "*is an element of.*"

$$2 \in \{1, 2, 3\}$$

The number 2 is also an element of the set of natural numbers, which can be designated N.

$$2 \in N$$

To show that 0 is *not* an element of set N, we draw a slash through the symbol $\in$.

$$0 \notin N$$

Two sets are equal if they contain exactly the same elements. For example,

$$\{1, 2\} = \{2, 1\}. \quad \text{Order does not matter.}$$

To indicate that two sets are not equal, we use the symbol $\neq$, which is read "*is not equal to.*" For example,

$$\{1, 2\} \neq \{0, 1, 2\}$$

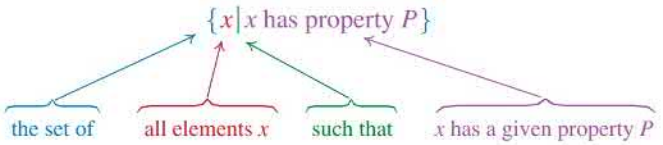
because one set contains the element 0 and the other does not.

A **variable** is a symbol, usually a letter, used to represent a number or to define a set of numbers. For example,

$$\{x \mid x \text{ is a natural number between 3 and 15}\}$$

(read "*the set of all elements x such that x is a natural number between* 3 *and* 15") defines the set $\{4, 5, 6, 7, \ldots, 14\}$.

The notation $\{x \mid x \text{ is a natural number between 3 and 15}\}$ is an example of **set-builder notation.**



EXAMPLE 1 **Listing the Elements in Sets**

List the elements in each set.

(a) $\{x \mid x \text{ is a natural number less than } 4\}$

(b) $\{x \mid x \text{ is one of the first five even natural numbers}\}$

(c) $\{x \mid x \text{ is a natural number greater than or equal to } 7\}$

SOLUTION

(a) The natural numbers less than 4 are 1, 2, and 3. This set, which is finite, is $\{1, 2, 3\}$.

(b) The set of the first five even natural numbers is $\{2, 4, 6, 8, 10\}$.

(c) The set of natural numbers greater than or equal to 7 is an infinite set, written

$$\{7, 8, 9, 10, \ldots\}.$$

✔ **Now Try Exercises 11 and 13.**

EXAMPLE 2 **Using Set-Builder Notation to Describe Sets**

Use set-builder notation to describe each set.

(a) $\{1, 3, 5, 7, 9\}$ **(b)** $\{5, 10, 15, \ldots\}$

SOLUTION

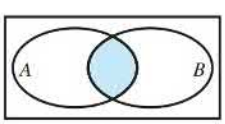
(a) One way to describe the set $\{1, 3, 5, 7, 9\}$ using set-builder notation is

$$\{x \mid x \text{ is one of the first five odd natural numbers}\}.$$

(b) The set $\{5, 10, 15, \ldots\}$ can be described as

$$\{x \mid x \text{ is a positive multiple of } 5\}.$$

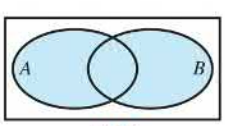
✔ **Now Try Exercises 23 and 25.**



$A \cap B$

Given two sets A and B, the set of all elements belonging to both set A ***and*** set B is the **intersection** of the two sets, written $A \cap B$. For example, if $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$, then

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}.$$



$A \cup B$

The set of all elements belonging to set A ***or*** to set B (or to both) is the **union** of the two sets, written $A \cup B$. For example, if $A = \{1, 3, 5\}$ and $B = \{3, 5, 7, 9\}$, then

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}.$$

A good way to picture a set of numbers is to use a **number line.** See **Figure 1.**

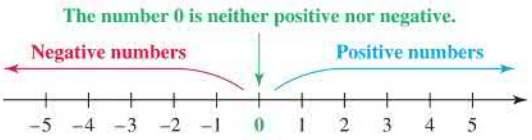


Figure 1

To draw a number line, choose any point on the line and label it 0. Then choose any point to the right of 0 and label it 1. Use the distance between 0 and 1 as the scale to locate, and then label, other points.

The set of numbers identified on the number line in **Figure 1,** including positive and negative numbers and 0, is part of the set of **integers.**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad \text{Integers}$$

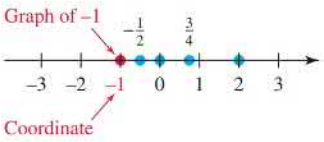


Figure 2

Each number on a number line is the **coordinate** of the point that it labels, and the point is the **graph** of the number. **Figure 2** shows a number line with several points graphed on it.

The fractions $-\frac{1}{2}$ and $\frac{3}{4}$, graphed on the number line in **Figure 2,** are examples of *rational numbers*. A **rational number** can be expressed as the quotient of two integers, with denominator not 0. The set of all rational numbers is written as follows.

$$\left\{\frac{p}{q} \;\middle|\; p \text{ and } q \text{ are integers, where } q \neq 0\right\} \quad \text{Rational numbers}$$

The set of rational numbers includes the natural numbers, whole numbers, and integers because these numbers can be written as fractions.

$$14 = \frac{14}{1}, \quad -3 = \frac{-3}{1}, \quad \text{and} \quad 0 = \frac{0}{1} \quad \text{Rational numbers}$$

A rational number written as a fraction, such as $\frac{1}{8}$ or $\frac{2}{3}$, can also be expressed as a decimal by dividing the numerator by the denominator.

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

← Terminating decimal (rational number)

← Remainder is 0.

$$\frac{1}{8} = 0.125$$

$$\begin{array}{r} 0.666\ldots \\ 3\overline{)2.000\ldots} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

← Repeating decimal (rational number)

← Remainder is never 0.

$$\frac{2}{3} = 0.\overline{6}$$

← A bar is written over the repeating digits(s).

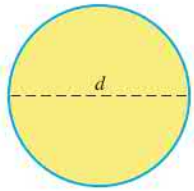
Thus terminating decimals, such as $0.125 = \frac{1}{8}$, $0.8 = \frac{4}{5}$, and $2.75 = \frac{11}{4}$, and decimals that have a repeating block of digits, such as $0.\overline{6} = \frac{2}{3}$ and $0.\overline{27} = \frac{3}{11}$, are rational numbers.

Decimal numbers that neither terminate nor repeat, which include many square roots, are **irrational numbers.**

$$\sqrt{2} = 1.414213562\ldots \quad \text{and} \quad -\sqrt{7} = -2.6457513\ldots \quad \text{Irrational numbers}$$

NOTE Some square roots, such as $\sqrt{16} = 4$ and $\sqrt{\frac{9}{25}} = \frac{3}{5}$, are rational.

A decimal number such as 0.010010001 . . . has a pattern, but it is irrational because there is no fixed block of digits that repeat. Another irrational number is π. See **Figure 3.**



π, the ratio of the circumference of a circle to its diameter, is approximately equal to 3.141592653....

$$\pi = \frac{C}{d}$$

Figure 3

Some rational and irrational numbers are graphed on the number line in **Figure 4.** The rational numbers together with the irrational numbers make up the set of **real numbers.**

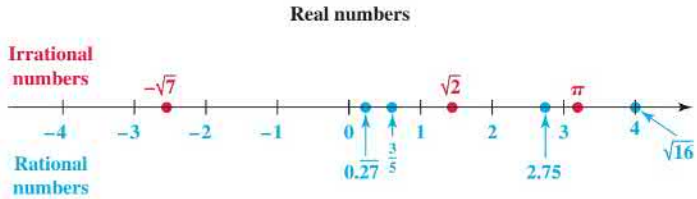


Figure 4

Every point on a number line corresponds to a real number, and every real number corresponds to a point on the number line.

The sets of numbers discussed so far are summarized below.

Sets of Numbers

Natural numbers, or counting numbers	$\{1, 2, 3, 4, \ldots\}$
Whole numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational numbers	$\left\{\frac{p}{q} \middle\| p \text{ and } q \text{ are integers, where } q \neq 0\right\}$ *Examples:* $\frac{4}{1}$, 1.3, $-\frac{9}{2}$ or $-4\frac{1}{2}$, $\frac{16}{8}$ or 2, $\sqrt{9}$ or 3, $0.\overline{6}$
Irrational numbers	$\{x \mid x$ **is a real number that cannot be represented by a terminating or repeating decimal**$\}$ *Examples:* $\sqrt{3}, -\sqrt{2}, \pi$, 0.010010001 . . .
Real numbers	$\{x \mid x$ **is a rational or an irrational number**$\}$*

* An example of a number that is not real is $\sqrt{-1}$. This number is part of the *complex number system* and is discussed later in the text.

Figure 5 shows the set of real numbers. ***Every real number is either rational or irrational.*** Notice that the integers are elements of the set of rational numbers and that the whole numbers and natural numbers are elements of the set of integers.

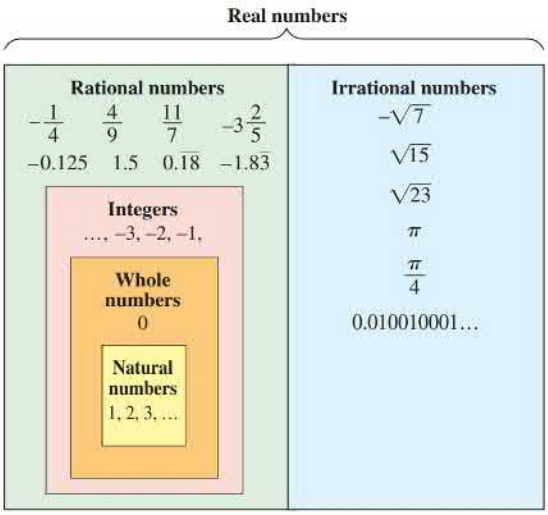


Figure 5

EXAMPLE 3 **Identifying Sets of Numbers**

Let $A = \left\{-8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6\right\}$. List all the elements of A that belong to each set.

(a) Natural numbers **(b)** Whole numbers **(c)** Integers

(d) Rational numbers **(e)** Irrational numbers **(f)** Real numbers

SOLUTION

(a) Natural numbers: 1 and 6 **(b)** Whole numbers: 0, 1, and 6

(c) Integers: $-8, -6, -\frac{12}{4}$ (or -3), 0, 1, and 6

(d) Rational numbers: $-8, -6, -\frac{12}{4}$ (or -3), $-\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1$, and 6

(e) Irrational numbers: $\sqrt{2}$ and $\sqrt{5}$

(f) All elements of A are real numbers. ✔ **Now Try Exercises 29, 31, and 33.**

EXAMPLE 4 **Determining Relationships between Sets of Numbers**

Determine whether each statement is *true* or *false*. If it is false, tell why.

(a) All irrational numbers are real numbers.

(b) Every rational number is an integer.

SOLUTION

(a) This statement is true. As shown in **Figure 5,** the set of real numbers includes all irrational numbers.

(b) This statement is false. Although some rational numbers are integers, other rational numbers, such as $\frac{2}{3}$ and $-\frac{1}{4}$, are not.

✔ **Now Try Exercises 37 and 41.**

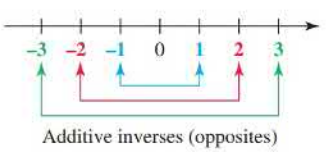


Figure 6

Additive Inverses In **Figure 6,** for each positive number, there is a negative number on the opposite side of 0 that lies the same distance from 0. These pairs of numbers are *additive inverses*, *opposites*, or *negatives* of each other. For example, 3 and -3 are additive inverses.